PAPER ID-421669

Subject Code: KAG201

Roll No: | | | | | | | | | | | | | |

BTECH
(SEM II) THEORY EXAMINATION 2021-22
BASIC MATHEMATICS II

Time: 3 Hours ***Total Marks: 100***

Notes:

- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief	Marks**(10X2=20)**	**CO**	**BL**
Q1(a)	State Cayley Hamilton theorem and verify for the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$		CO1	KL1
Q1(b)	If $A = \begin{bmatrix} 1-i & 2i \\ 3+5i & 5 \end{bmatrix}$, find the transposed conjugate of A.		CO1	KL5
Q1(c)	Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic.		CO2	KL3
Q1(d)	Show that the function $f(z) = \lvert z\rvert^2$ where $z = x + iy$ is analytic only at origin.		CO2	KL3
Q1(e)	Find the curl and divergent of the vector function$\vec{F} = xy\hat{\imath} + y^2z\hat{\jmath} - xz^3\hat{k}$ at point $(1,-1,0)$.		CO3	KL5
Q1(f)	Find the unit normal vector to the surface $z^2 = x^2 + y^2$ at the point $(2,1,1)$.		CO3	KL5
Q1(g)	Write the Dirichle's conditions for a Fourier series.		CO4	KL1
Q1(h)	Find the partial differential equation of the equation $z = f(x^3 - y^3)$.		CO4	KL5
Q1(i)	Classify the partial differential equation $y^2r - 2xys + x^2t = \frac{y^2}{z}p + \frac{x^2}{z}q$.		CO5	KL4
Q1(j)	Write two dimensional heat flow equations for steady state and transient state.		CO5	KL1

SECTION-B	Attempt **ANY THREE** of the following Questions	Marks**(3X10=30)**	**CO**	**BL**
Q2(a)	Show that the equations $-2x + y + z = a$ $x - 2y + z = b$ $x + y - 2z = c$ Have no solution unless $a + b + c = 0$. In which case they have infinitely many solutions? Find these solutions when $a = 1, b = 1$ and $c = -2$.		CO1	KL3
Q2(b)	Show that the function $f(z)$ defined by$f(z) = \begin{cases} \frac{x^3y^5(x+iy)}{x^6+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at the origin even though it satisfies Cauchy Riemann equations at the origin.		CO2	KL3
Q2(c)	If a vector field is given by $\vec{V} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$. Is this field irrotational? If so, find its scalar potential.		CO3	KL5
Q2(d)	Find the Fourier series of the function$f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$		CO4	KL5
Q2(e)	A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. if the temperature along one short edge $y = 0$ is given by $u(x,0) = 100\sin\frac{\pi x}{8}$, $0 < x < 8$. while the tow long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at 0° C, show that the steady state temperature at any point of the plate is given by $u(x,y) = 100e^{-\frac{\pi y}{8}}\sin\frac{\pi x}{8}$.		CO5	KL3

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SECTION-C	Attempt **ANY ONE** of the following Questions	Marks **(1X10=10)**	CO	BL
Q3(a)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$		CO1	KL5
Q3(b)	Find the rank of the matrix $A = \begin{bmatrix} 3 & 4 & -3 \\ 6 & 1 & 2 \\ 7 & -5 & 0 \end{bmatrix}$ by reducing A into normal form.		CO1	KL5

SECTION-C	Attempt **ANY ONE** of the following Questions	Marks **(1X10=10)**	CO	BL
Q4(a)	Show that $e^x(xcos\ y - ysin\ y)$ is a harmonic function. Find he analytic function for which $e^x(xcos\ y - ysin\ y)$ is imaginary part.		CO2	KL5
Q4(b)	If $f(z)$ is analytic function of z, prove that: $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\|Ref(z)\|^2 = 2\|f'(z)\|^2$.		CO2	KL3

SECTION-C	Attempt **ANY ONE** of the following Questions	Marks **(1X10=10)**	CO	BL
Q5(a)	Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point P $(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$.		CO3	KL5
Q5(b)	If $\vec{a}$ is constant vector and $\vec{r}$ is the radius vector, prove that (i) $div\ (\vec{r} \times \vec{a}) = 0$ (ii) $curl\ (\vec{r} \times \vec{a}) = -2\vec{a}$.		CO3	KL3

SECTION-C	Attempt **ANY ONE** of the following Questions	Marks **(1X10=10)**	CO	BL
Q6(a)	Solve the partial differential equation $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} - 4\frac{\partial^3 z}{\partial y^3} = 4\sin(2x + y) + e^{x+2y}$		CO4	KL3
Q6(b)	Find the real function V of x and y, reducing to zero when $y = 0$ and satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$.		CO4	KL5

SECTION-C	Attempt **ANY ONE** of the following Questions	Marks **(1X10=10)**	CO	BL
Q7(a)	Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial x} + 2u$.		CO5	KL3
Q7(b)	A string is stretched and fastened to two points *l* apart. Motion is started by displacing the string in the form $y = A\sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x, t) = A\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l}$.		CO5	KL3

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