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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 39912 Roll No.

B. Tech. Examination 2023-24

(Odd Semester)

COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : 8×1=8

   (a) Define an analytic function.

   (b) Find the value of m, if the function

   $$2x - x^2 + my^2$$

   be harmonic.

   (c) State Liouville theorem.

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(d) Define an isolated singularity of the function with example.

(e) Find the Laplace transform of $(t-1)\,U(t-1)$.

(f) Find the inverse Laplace transform of $\frac{1}{s^{1/2}}$.

(g) State the Fourier integral theorem.

(h) Write the Z-transform of a sequence $\{f(k)\}$.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z by Milne-Thomson method.

(b) Evaluate :

$$\int_0^{2\pi} \frac{1}{5-4\sin\theta}\, d\theta$$

by contour integration in the complex plane.

(c) Solve :

$$y'' + 4y' + 4y = 6e^{-t}$$

$$y(0) = -2,\ y'(0) = 8$$

using Laplace transform method.

(d) Find the inverse Z-transform of :

$$\frac{1}{(z-3)(z-2)}$$

(i) $2 < |z| < 3$

(ii) $|z| > 3$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question. $5 \times 8 = 40$

3. (a) Using the Cauchy-Riemann equations, show that $f(z) = z^3$ is analytic in the entire plane.

(b) Find the value of the integral :

$$\int_0^{1+i} \left(x - y + ix^2\right) dz$$

from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$.

(c) Use Cauchy integral formula to evaluate :

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

Where C is the circle $|z| = 3$.

4. (a) Find the first form terms of the Taylor's series expansion of :

$$f(z) = \frac{z+1}{(z-3)(z-4)}$$

abou $z = 2$.

(b) Expand the function :

$$f(z) = \frac{1}{z(z-1)(z-2)}$$

in a laurent series.

(c) Using residue theorem, evaluate :

$$\int_C \frac{z^2}{(z-1)^2(z+2)}\,dz$$

where C is $|z| = 3$.

5. (a) Find the Laplace transform of :

$$\frac{\cos at - \cos bt}{t}$$

(b) Find the inverse Laplace transform of :

$$\frac{s}{\left(s^2 + a^2\right)^2}$$

(c) Use Laplace transform to solve :

$$\frac{dx}{dt} + 4y =, \quad \frac{dy}{dt} - 9x = 0$$

$x(0) = 2, y(0) = 1.$

6. (a) Express the function :

$$f(x) = \begin{cases} 1 & \text{when } |x| \leq 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

as a Fourier integral. Hence evaluate :

$$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$$

(b) Find the Fourier cosine transform of :

$$f(x) = 5\, e^{-2x} + 2\, e^{-5x}$$

(c) Find the Z-transform of $\{f(k)\}$, where :

$$f(k) = \begin{cases} 5^k, & K < 0 \\ 3^k, & K \geq 0 \end{cases}$$

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