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Following Paper ID and Roll No. to be filled in your Answer Book.

PAPER ID : 39906 Roll No.

B. Tech. Examination 2022-23

(Special Carry Over Paper)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : 8×1=8

(a) Solve :

$$(D^4 - 1)\, y = 0$$

(b) Find the particular integral of :

$$(D^2 - 4D + 4)\, y = x^3 e^{2x}$$

(c) Evaluate :

$$\int_{-1}^{1} P_3^2 (n)\, dx$$

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(d) Prove that :

$$\frac{d}{dx}[J_0(n)] = -J_1(n)$$

(e) If $f(n) = 1$ is expanded in a fourier sine series in $(0, 2)$ then find the value of b_n.

(f) Find the constant term if $f(n) = 2x^3 - x^2$ is expanded in fourier series defined in $(-\pi, \pi)$.

(g) Classify the following partial differential equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(h) Write the Laplace equation in two dimensions.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Use variation of parameter method to solve :

$$\frac{d^2y}{dx^2} + y \tan x$$

(b) Find the power series solution of :

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0 \text{ about } x = 0$$

(c) Find the fourier series for $f(n)$ if :

$$f(n) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Also deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$

(d) If a string of length ℓ is initially at rest in equilibrium position and each of its points is given the velocity :

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3\left(\frac{\pi x}{t}\right)$$

find the displacement $y(x, t)$.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each question. 5×8=40

3. (a) Solve :

$$(1+x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos[\log(1+x)]$$

(b) Solve the simultaneous differential equation :

$$\frac{dx}{dt} = 3x + 8y$$

$$\frac{dy}{dt} = -x - 3y$$

(c) Solve :

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + \left(4x^2 - 1\right) y = -3e^{x^2} \sin 2x$$

by removing the first derivative.

4. (a) Prove that :

$$J_{-½}(n) = \sqrt{\frac{2}{\pi x}} \cos n$$

(b) Prove that :

$$\int_{-1}^{1} \left[P_n(n)\right]^2 dx = \frac{2}{2n+1}$$

(c) Prove that :

$$(x+1)\, P_{n+1}(n) = (2n+1)\, x\, P_n(n) - n\, P_{n-1}(n)$$

5. (a) Find the half range sine series expansion of :

$$f(n) = x^2 \text{ in } 0 < x < 3$$

(b) Find the fourier series expansion for the function :

$$f(n) = x \sin x,\ 0 < x < 2\pi$$

(c) Find fourier half range cosine series of the function :

$$f(n) = \begin{cases} 2x, & 0 < x < 1 \\ 2(2-x), & 1 < x < 2 \end{cases}$$

6. (a) Solve the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\, \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$$

(b) Solve the partial differential equation :

$$(D^2 - DD' - 2D)\, z = x^2 y$$

(c) Solve :

$$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$$

using method of separation of variable.

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