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BAS 3201 / NBS 4201

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Following Paper ID and Roll No. to be filled in your Answer Book

PAPER ID : 49905 Roll No.

B. Tech. Examination, 2026

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : 8×1=8

(a) Find the order and degree of the differential equation :

$$\left(\frac{d^3y}{dx^3}\right)^2 - \left(\frac{dy}{dx}\right)^3 = 1$$

(b) Find the particular integral of the differential equation :

$$(D^2 - 1)\, y = 5$$

[P. T. O.

(c) Show that $x = 0$ is not an ordinary point of the differential equation :

$$2x^2 y'' - x y' + (x - 5) y = 0$$

(d) Find the value of :

$$\int_{-1}^{1} P_5^2(x)\, dx$$

(e) Determine the fourier coefficient 'a_0' of a function x^2 in the interval (0, 4).

(f) If the function $f(x)$ is expanded in fourier series in $(-k, k)$ then write the constant term.

(g) Classify the following partial differential equation $Z_{xx} = Z_{yy}$.

(h) Solve :

$$\frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$$

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Solve :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1 + e^x}$$

by method of variation of parameters.

(b) Solve the following differential equation in series :

$$(1 - x^2) y'' - 2x y' + 2y = 0$$

(c) Find the fourier series expansion for :

$$f(x) = x + \frac{x^2}{4}, -\pi \le x \le \pi$$

(d) Find the deflection $u(x, t)$ of a tightly stretched vibrating string of unit length that is initially at rest and whose initial position is given by :

$$\sin \pi x + \frac{1}{3}\sin(3\pi x) + \frac{1}{5}\sin(5\pi x),\ 0 < x < 1$$

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. 8×5=40

3. (a) Solve :

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = \sin 3x$$

(b) Solve the following system of differential equations :

$$D_x + D_y + 3x = \sin t$$

$$D_x + y - x = \cos t$$

[P. T. O.

(c) Solve :

$$y'' - 4xy' + (4x^2 - 1)y = 3e^{x^2} \sin 2x$$

4. (a) Prove that :

$$P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$$

(b) Prove that :

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

(c) Express $J_5(x)$ in terms of $J_1(x)$ and $J_2(x)$.

5. (a) Find the fourier series of the function :

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$

(b) Expand $f(x) = x$ as a half range cosine series in $(0, 2)$.

(c) Find a fourier series to represent $f(x) = \pi - x$ for $0 < x < 2\pi$.

6. (a) Solve :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$$

(b) Solve :

$$(D - 3D' - 2)^2 z = 2e^{2x} \sin(y + 3x)$$

(c) Solve :

$$\frac{\partial u}{\partial x} = 3\frac{\partial u}{\partial t}$$

using method of seperation of variables.

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