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B. Tech. Examination, 2024-25

(Even Semester)

DIFFERENTIAL EQUATIONS AND FOURIER ANALYSIS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION–A

1. Attempt all parts of the following : 8×1=8

(a) Solve $(D^3 + 2D^2 - 3D)y = e^t$, where $D = \frac{d}{dt}$.

(b) Solve simultaneous differential equations :

$$\frac{dx}{dt} = 3y, \frac{dy}{dt} = 3x$$

(c) Show that :

$$P_n(-x) = (-1)^n P_n(x)$$

(d) Prove that :

$$J_{-n}(x) = (-1)^n J_n(x)$$

(e) Find the fourier constant a_1 of :

$$f(x) = \pi x, -\pi \leq x \leq \pi$$

(f) Find the fourier constant a_0 for the function :

$$f(x) = \begin{cases} 0, & -\pi < r < 0 \\ x, & 0 < x < \pi \end{cases}$$

(g) Solve the partial differential equation :

$$\frac{\partial z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = 0$$

(h) How many arbitrary constants in the solution of Laplace equation?

SECTION – B

2. Attempt any two parts of the following : $2 \times 6 = 12$

(a) Using method of variation of parameter solve :

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{2x} + x^2$$

(b) Show that :

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = 0, \quad m \neq n$$

(c) Find half range cosine series for the function $f(x) = (x-1)^2$ in the interval $(0, 1)$. Hence prove that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$$

(d) Solve by the method of separation of variables :

$$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u, \quad u = 3e^{-x} - e^{-5x}$$

when $t = 0$.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. $8 \times 5 = 40$

3. (a) Solve the simultaneous differential equation :

$$\frac{dx}{dt} + 5x - 2y = t, \quad \frac{dy}{dt} + 2x + y = 0$$

being given $x = 0$ and $y = 0$ when $t = 0$.

(b) Solve by changing the independent variable :

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$

(c) Solve :

$$(D^2 - 3D + 2)y = x^2 + 2x + 1$$

4. (a) Solve in series the differential equation :

$$2x(1-x)\frac{d^2y}{dx^2}+(5-7x)\frac{dy}{dx}-3y=0$$

(b) Prove that :

$$\frac{1-z^2}{(1-2xz+z^2)^{3/2}}=\sum_{n=0}^{\infty}(2n+1)P_n(x)z^n$$

(c) Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

5. (a) Find the fourier series of :

$$f(x)=\begin{cases} x, & 0<x<1 \\ 1-x, & 1<x<2 \end{cases}$$

(b) Expand $f(x) = 2x - 1$ as a sine series in $0<x<\pi$.

(c) Find the fourier series of $f(x) = x^2$ in $-\pi \leq x \leq \pi$.

6. (a) A rod of length ℓ with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to $0\ ^0C$ and are kept at that temperature. Calculate the temperature function $u(x, t)$.

(b) Determine the solution of one dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

under the conditions $u(0, t) = 0$, $u(\ell, t) = 0$ for all t, $u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$, $0 < x < \ell$.

(c) Solve :

$$(D + 1)(D + D' - 1) z = \sin(2x + 3y)$$

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