B. Tech. Examination, 2025-26

(Special Carry Over Paper)

MATRICES AND CALCULUS

Time : Three Hours] *[Maximum Marks : 60*

Note :– Attempt all questions.

SECTION – A

1. Attempt all parts of the following : $8 \times 1 = 8$

(a) Define identity matrix of order 2×2.

(b) Write the statement of Cayley-Hamilton theorem.

(c) Evaluate $D^n (e^{ax})$.

(d) Define :

$$\frac{\partial (u, v)}{\partial (x, y)}$$

[P. T. O.

vo V25 Pro
1, 2026, 06:48

(e) What do you meanby double integral?

(f) Write the value of $\lfloor \frac{1}{2}$.

(g) If $\vec{r} = t^2\,\hat{i} + 2t\,\hat{j} + \hat{k}$, then find the value of $\frac{d\vec{r}}{dt}$.

(h) Write the statement of Gauss divergence theorem.

SECTION – B

2. Attempt any two parts of the following : 2×6=12

(a) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

(b) If $y = \sin(m \sin^{-1} x)$, prove that :

$$(1-x^2)\,y_{n+2} - (2n+1)\,x - y_{n+1} + (m^2 - n^2)\,y_n = 0$$

(c) Evaluate :

$$\int_0^1 \int_0^y x\,y\,e^{-x^2}\,dx\,dy$$

(d) Use Green's theorem to evaluate :

$$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$$

where C is the square formed by the lines $y = \pm 1, x = \pm 1$.

SECTION – C

Note :– Attempt all questions. Attempt any two parts from each questions. 8×5=40

3. (a) Determine for what values of λ and μ the following equations :

$$x + y + z = 6,$$
$$x + 2y + 3z = 10,$$
$$x + 2y + \lambda z = \mu$$

have :

(i) No solution

(ii) A unique solution

(iii) Infinite number of solutions

[P. T. O.

vivo V25 Pro
Nov 1, 2026, 06:48

NBS 4101

4

(b) Find the eigen values and corresponding eigen vectors for the matrix :

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

(c) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

and hence find A^{-1}.

4. (a) If :

$$u = \log\left(\frac{x^2 + y^2}{x + y}\right)$$

prove that :

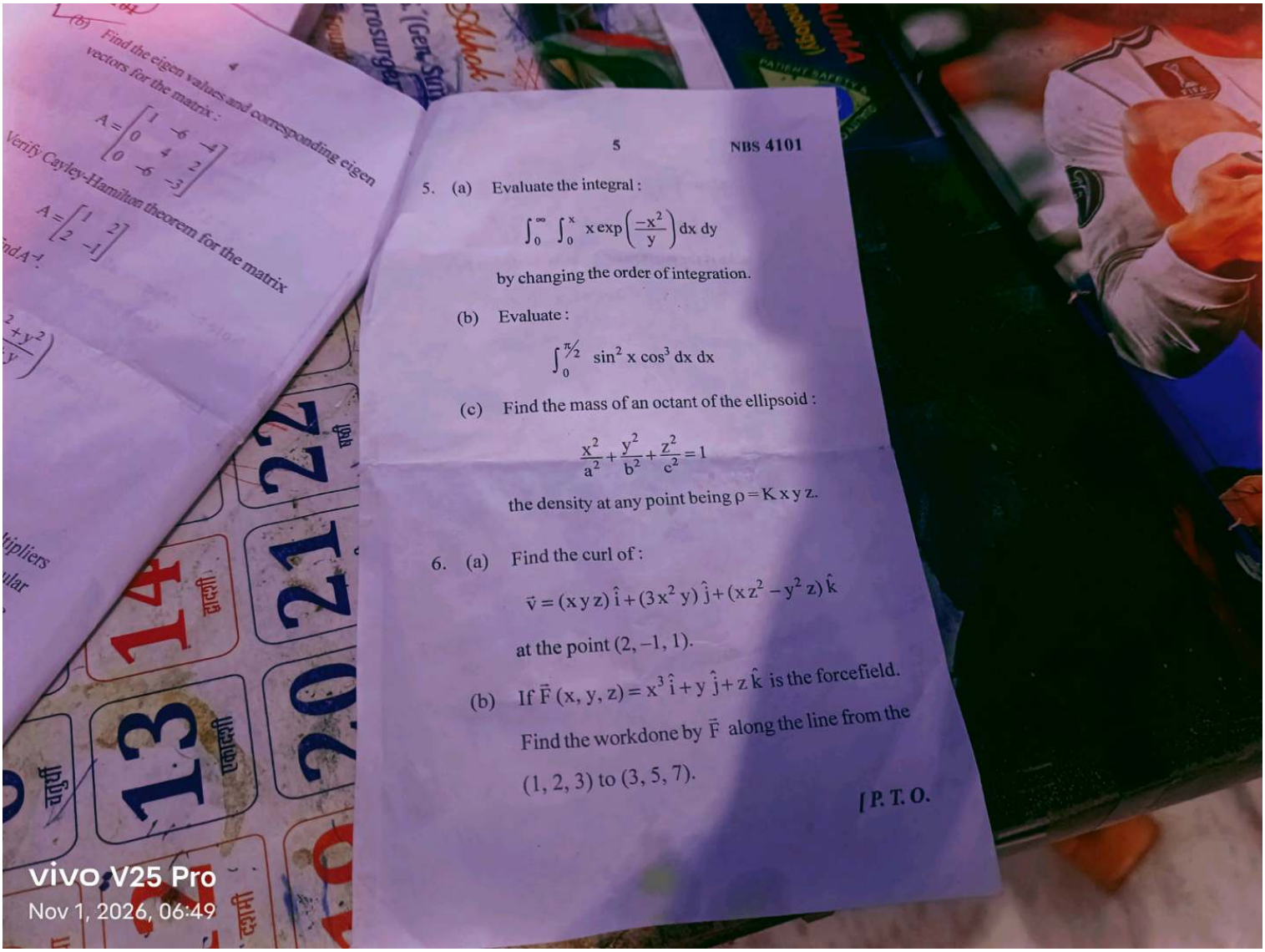
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$

(b) Expand sin x in power series.

(c) Use the method of the Lagrange's multipliers to find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

vivo V25 Pro
Nov 1, 2026, 06:49



5. (a) Evaluate the integral :

$$\int_0^{\infty} \int_0^{x} x \exp\left(\frac{-x^2}{y}\right) dx\, dy$$

by changing the order of integration.

(b) Evaluate :

$$\int_0^{\pi/2} \sin^2 x \cos^3 dx\, dx$$

(c) Find the mass of an octant of the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

the density at any point being $\rho = K x y z$.

6. (a) Find the curl of :

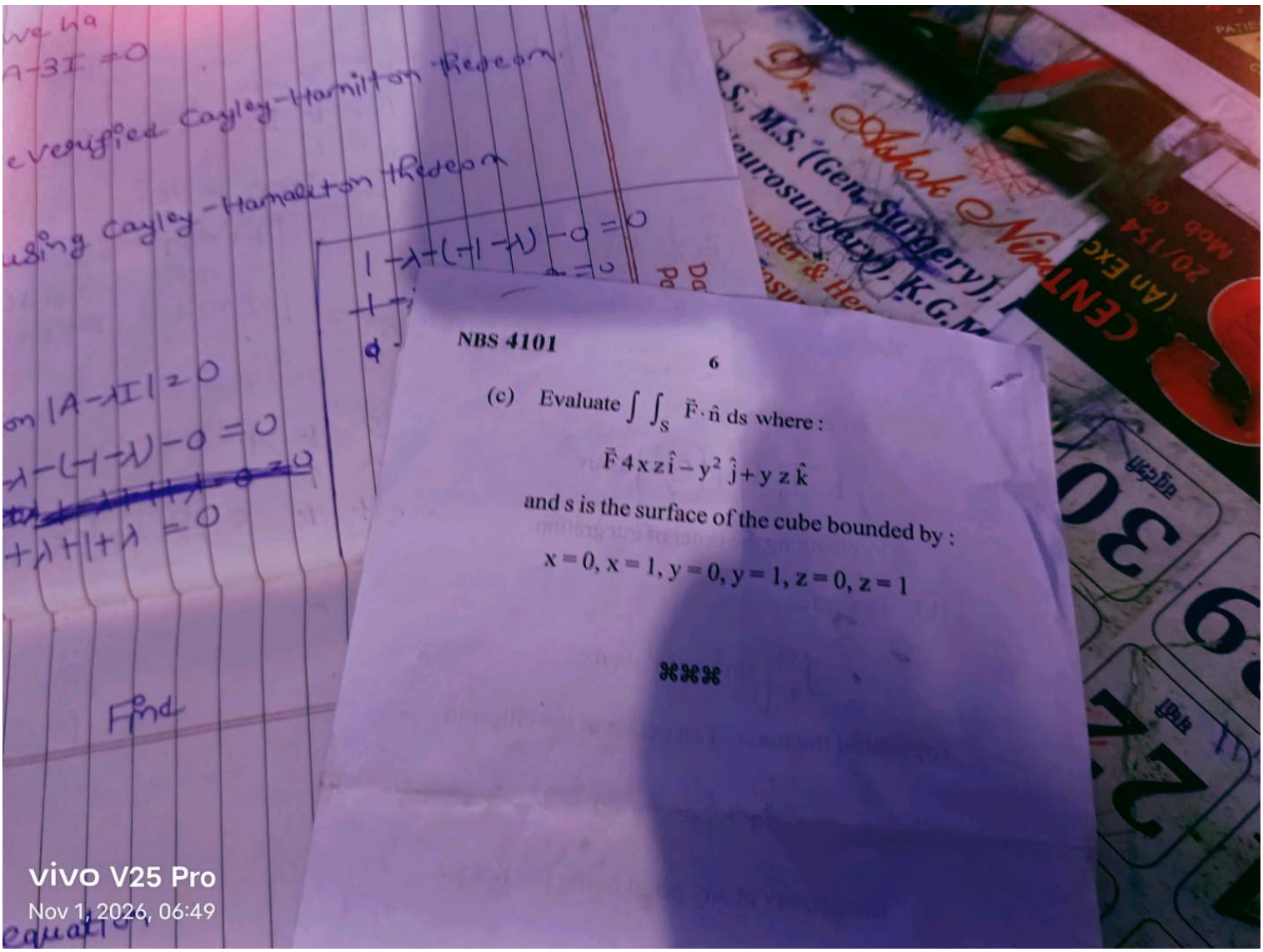
$$\vec{v} = (x y z)\,\hat{i} + (3x^2 y)\,\hat{j} + (x z^2 - y^2 z)\,\hat{k}$$

at the point (2, –1, 1).

(b) If $\vec{F}(x, y, z) = x^3\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ is the forcefield. Find the workdone by $\vec{F}$ along the line from the (1, 2, 3) to (3, 5, 7).

[P. T. O.

vivo V25 Pro
Nov 1, 2026, 06:49



(c) Evaluate $\int\int_S \vec{F}\cdot\hat{n}\,ds$ where :

$$\vec{F}\,4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$$

and s is the surface of the cube bounded by :

$$x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$$

⌘⌘⌘

vivo V25 Pro
Nov 1, 2026, 06:49